Nonparametric Estimation of Expected Shortfall

ON X C

Io St te niversity

ABSTRACT e e pec ed o f n nc e n y pop e e n n nc n e en nd po e de ed dd x p ope y c n fo e e e con de o nonp e c e pec ed o f e o fo dependen n nc o One p e e e of e ce e o e n e e e ne o ed e on of e o c e *M the tic Fin nce* op n o e cc e e on c n e e ed y o n O n y e e e e e e ne o n doe no p od ce o e cc e e on of e o f d e en f e e e ne n e o f

1. INTRODUCTION

e e pec ed o f nd e e e pop e e of n nc fo n o po fo o of A zne De en nd e o c e dd y p ope y n ene nd ence no de e en e e n con de e en o e nd ed nd eco e o e c e e n e n n nc n e en

Le $\{X_t\}_{t=1}^{n}$ e e e e of n o po fo o of o e n pe od of e n Le Y_t $-log\ X_{it}/X_{it-1}$ e ne e o e n o o o e e t pe od ppo $\{Y_t\}_{j=1}^{n}$ dependen on y p oce n d on f nc on F en po e e p c o o ze o e con dence e e $-p$

$$\nu_p \quad \mathrm{nf}\{y \quad F\ y \geq \quad -p\}$$

e $-p$ q n e of e o d on F e pec e e e of

nonp e c ene e nd pp e fo n y n y n e con of
po fo o oc on

An d n e of e nonp e c od e n ode f ee n ence ode
o nd o d c d y n pec ed o d on n nc n
en p y conce ned c of p of e o d on
o e e d e ene y p n e n ence p opo n p ope p e c o
ode deq e fo e p no e e e nonp e c
od c n p y n c n o e An e d n e of e nonp e c pp o
o de n e of d dependence e d p e n e con
of n nc o e nonp e c e con de ed n p pe c n cc o
d e d dependence e p c y nce e e ec of e dependence on e nce of e
[redacted] e on c n e c e y pe o n e nce fo d e en f e
e e e pp o e e e ec e y e e ceed nce ndependen nd
den c y d on o on e p y nde e ed D nd
D' cond on Le d e L nd en nd oo zen An p c dy y Be n nd
nc y c y n o nonp e c n e o n n nc
e n c n on dependence e en fo e o d e e nd c e e
need fo con de n e dependence n n nc e n d ec y e pp o en
y e nonp e c e con de ed n p pe

n p pe e e e o nonp e c [redacted] e One d on e ed
p e e e of e ce e o de ned y e ν_p d on n o de c
An e e ne e p opo d n c e e p oy e ne o n
n e n e on nd e n e n of e e ce e o ope
e e ne o n o d e e on o e cc e o ed fo e c
of e on y en nd n o e e o n y e e nce
nd e e n e e o of e e ne e no nece y e n of
p e e e e A e fo e on of e [redacted] e p e p e e e
of e ce e o ood eno nd e e no need c y o e ne o n
y e p n con de n e ne o n e d e nce n q n e
e on fo ndependen e e nd M on nd dependen en nd

n 4 o on nde y n e n fo o n n e d e en e ec of e ne o n y on e f c e ec e y e n p e e c n e e ed e cc e y y p e e n

e p pe c ed fo o e n od ce e o nonp e c e n ec on e c p ope e d e d n ec on ec on epo on e fo o ed y n e p c dy on o n nc e n ec on A e n c de e en n e ppend

2. NONPARAMETRIC ESTIMATORS

A p e nonp e c e of e e ed e e of e ce e o e n ν_p e e ν_p $Y_{([n(1-p)]+1)}$ e pe q n e of ν_p nd $Y_{(r)}$ e o de c of $\{Y_t\}_{t=1}^n$ ence

$$\mu_p \quad \frac{\sum_{t=1}^n Y_t I \; Y_t \geq \nu_p)}{\sum_{t=1}^n I \; Y_t \geq \nu_p)} \qquad np)^{-1} \sum_{t=1}^n Y_t I \; Y_t \geq \nu_p) \qquad (1)$$

e e I ·) e nd c f nc on

e e ne e p opo d y c e e fo o n Le K e e ne f nc on y den y f nc on nd $G\; t)$ $\int_t^\infty K\; u)du$ nd $G_h\; t)$ $G\; t/$) e e po e o n nd e e ne e of e f nc on $S\; x)$ $- F\; x)$

$$S_h\; z) \quad n^{-1} \sum_{t=1}^n G_h\; z - Y_t) \qquad (2)$$

A e ne e of ν_p deno ed $\nu_{p,h}$ e on of $S_h\; z)$ p p opo d n o e o L en nd c e)[2] By ep c n e nd c f nc on nd ν_p e o e G_h nd $\nu_{p,h}$ e pec e y n) c e) p opo d e fo o n e ne e

$$\mu_{p,h} \quad np)^{-1} \sum_{t=1}^n Y_t G_h\; \nu_{p,h} - Y_t) \qquad (3)$$

[2] Its statistical properties and how to obtain the standard errors are considered in Chen and Tang (2005). See also Cai (2002) and Fan and Gu (2003) for ke

B d on e p o e en of e e ne e $\nu_{p,h}$ o e ν_p e pec ed e e ne [redacted] e $\mu_{p,h}$ o d p o e e e on cc cy of e n ed e μ_p Con n o e e foc of e ne c on

3. MAIN RESULTS

e p ope e of e o nonp e c [redacted] e e e ed n c on e e cond on

Le $\mathcal{F}_k^l$ e e σ e of e en ene ed y $\{Y_t, k \le t \le l\}$ fo $l > k$ e α n coe c en n od ced y o n

$$\alpha\ k \quad \text{p}_{\mathcal{A}\in\mathcal{F}_1^i, \mathcal{B}\in\mathcal{F}_{i+k}^\infty} \ |P\ AB - P\ A P\ B|.$$

e e d e α n f $_{k\to\infty}\alpha\ k$ e dependence de ed y e α n e e e p ed y e ype of n e Do n fo co p en e d on e fo o n cond on e ed n o dy

e e e $\rho \in$, $\alpha\ k \le C\rho^k$ fo $k \ge$ nd po e con n C

e d on F of Y_t e y con n o p o y den y f con n o cond de e n $\mathcal{B}\ \nu_p$ ne ood of ν_p e o n d on f nc on of $Y_1, Y_{k+1} F_k$ fo $k \ge$ e cond p de e o nded n $\mathcal{B}\ \nu_p$ $E\ |Y_t|^{2+\delta} \le C$ fo e $\delta >$ nd po e con n C

K e c p o y den y y n e en cond on $\int_{-1}^{1} u K\ u du$ nd $\int_{-1}^{1} u^2 K\ u du\ \ \sigma_K^2 >$ nd K o nded nd L p z con n o de e e $\to$ $n^{3-\beta} \to \infty$ fo ny $\beta >$ nd n o $n \to$ $n \to \infty$

Cond on e n e e e e e c α n ed y ny co on y d n nc e e nc de e A MA A C e c o y nd d on ode o n nce M y nd e e ed e α n fo A C ode nd enon C o e e nd L edo fo d on ode Cond on con n nd d cond on eq e nde y n o ne fo e n nd p o n den e e n en fo e e e n Cond on nd e e one eq ed y e e ne e e

Cond on eq en on e ne Cond on pec e n e fo e nd nc de $O(n^{-1/3})$ e op o de fo e n e on e cond on e co p e cond on po d y e o o n nce Cond on nd co e pond A p on nd f A p on nd c nd of c e e pec e y

Le $\gamma(k) = Cov\{(Y_1 - \nu_p)I(Y_1 \geq \nu_p), (Y_{k+1} - \nu_p)I(Y_{k+1} \geq \nu_p)\}$ fo po e n e e k nd

$$\sigma_0^2(p, n) = \{Var\{(Y_1 - \nu_p)I(Y_1 \geq \nu_p)\} + \sum_{k=1}^{n-1} \gamma(k)\}.$$

A p on nd e D ydo neq y p y $\sigma_0^2(p, n)$ n fo e n nd con e n $n \to \infty$

e e n e n o ed e μ_p p o de po n of efe ence fo e e ne e De on en n e ppend o nde cond on nd nd fo n y po e κ

$$\hat{\mu}_p - \mu_p = p^{-1}\{n^{-1}\sum_{i=1}^{n}(Y_t - \nu_p)I(Y_t \geq \nu_p) - p(\mu_p - \nu_p)\} + o_p(n^{-3/4+\kappa})$$

B d ype e p n on e d e fo o n eo e d n e p no y of $\hat{\mu}_p$

nde cond on $n \to \infty$

$$\sqrt{pn}\,\sigma_0^{-1}(p, n)(\hat{\mu}_p - \mu_p) \xrightarrow{d} N(0, 1).$$

e eo e nd c e p nce of $\hat{\mu}_p$ $\sigma_0^2(p, n)/(np)$ e nce of $p^{-1}\{n^{-1}\sum_{i=1}^{n}(Y_t - \nu_p)I(Y_t \geq \nu_p) - p(\mu_p - \nu_p)\}$ e d n o de n e e p n on e dependence n e o n e e e ec d n e p nce o e co nce n $\sigma_0^2(p, n)$ e n e need cc od e dependence n f e nfe ence n e on nd ype n fo e o f e on e no e e ec e p e ze fo e on on y np A p n n e een nd on y d n n nc n en e e ec o y co on en e fo nfe ence of e e e fo o n eo ze e p ope e of e e ne e , 4

nde cond on 4 , 4 $n \to \infty$

$$\sqrt{pn}\sigma_0^{-1} \; p. \; n4 \; \mu_{p,h} - \mu_p 4 \xrightarrow{d} N \quad , \; 4 \qquad 4$$

nd f e o e

$$Bias \; \mu_{p,h} 4 \quad -\tfrac{1}{2} p^{-1} \sigma_K^2 \; {}^2 f \; \nu_p 4 \quad o \quad {}^2 4 \quad \text{nd} \qquad 4$$

$$Var \; \mu_{p,h} 4 \quad p^{-1} n^{-1} \sigma_0^2 \; p. \; n4 \quad o \; n^{-1} \; 4 \qquad 4$$

By co p n eo fo nd e ene e p. c no d on n o ed p e e μ_p e co e pond n e fo e on epo ed n en nd n 4 e no e μ_p nd $\mu_{p,h}$ con e e μ_p e e of $\sqrt{n}$ o o e p ec y e e of $\sqrt{np}$. e e e e ν_p nd $\nu_{p,h}$ con e e ν_p e e of $\sqrt{n}$ o o e p ec y e e of $\sqrt{n} f \; \nu_p 4$ e e f e p o y den y of Y_t

e cond p of eo con ey d e en y f e on of n e e on e ne e doe no o e nce ed c on e cond o de of n^{-1} e cond o de n e A e e e o n n n n e d no e nce e n e n e e o e efo e fo e p po of e n e e ne o n co n e p od c e e nde y n e n e f c e ec e y e n p e e c n e e ed e cc e y o o n e on nonp e c e on of e e n p e e c n e e e ed y e p e e n

o d e no ed e o e en on y pp c e fo po n e on of o con c n con dence n e nd e n ype e on μ_p n e p e nce of d dependence e e ne o n c n p y n c n o e n e n $\sigma_0^2 \; p. \; n4$ e pec den y e on pp o

4. SIMULATION STUDY

n c on e epo e f on dy e e e pe fo nce of e nonp e c e no ec e con o eo e nd n n e p e o c on e ode o en fo e o o Y_t n e on e

n

e ne e on of e pec den y of

p $n/$ $q)$ nd $\sigma^2(q)$ $\max_{0\le j\le 2q-1} E\left(\sum_{l=[jp]+1}^{[(j+1)p]} X_l\right)^2$ n eq y en n Yo oy $\sigma^2(q) \le Cp$ App y eo e n Bo $)$ fo α n q ence

$$P\{|F_n(\nu_p + \epsilon_n) - F(\nu_p + \epsilon_n)| > C_1\epsilon_n\}$$
$$\le 4\exp\left(-\frac{C_1^2\epsilon_n^2 q}{\sigma^2(q)}\right) + \{\frac{}{C_1\epsilon_n}\}^{1/2} q\alpha\{n/(q)\} \qquad \text{A}\ )$$

e e $\sigma^2(q)$ $p^{-2}\sigma^2(q)$ ϵ_n $C\epsilon_n$ o o

$$4\exp\left(-\frac{C_1^2\epsilon_n^2 q}{\sigma^2(q)}\right) \le 4\exp\{-C_2\epsilon_n q\} \qquad \text{A}\ )$$

e e $C_2 >$ nce $n\epsilon_n^2 \to \infty$ e n $q\epsilon_n \to \infty$ e e n A) con e e q ze o e ponen y f On e cond e of A) e e e c α n p e

$$\{\left(\frac{}{C_1\epsilon_n}\right)^{1/2} q\alpha\{n/(q)\} \le C\epsilon_n^{-1/2} q\rho^{[n^{1/2}\log^{-1}(n)/2]} \qquad \text{A}\ )$$

e con e e q ze o e ponen y f oo co p e e e p oof of Le □

L MMA nde e cond on)) nd fo ny $\kappa >$

$$n^{-1}\sum(Y_t - \nu_p)\{I(Y_t \ge \nu_p) - I(Y_t \ge \nu_p)\} = o_p(n^{-3/4+\kappa})$$

OO Le W_t $(Y_t - \nu_p)\{I(Y_t \ge \nu_p) - I(Y_t \ge \nu_p)\}$ e e e $E(W_t)$ No e $E(W_t)$ $-I_{t1}$ I_{t2} e e

$$I_{t1} = E\{(Y_t - \nu_p)I(\nu_p \le Y_t < \nu_p)I(\nu_p > \nu_p)\} \quad \text{nd}$$
$$I_{t2} = E\{(Y_t - \nu_p)I(\nu_p \le Y_t < \nu_p)I(\nu_p < \nu_p)\}.$$

e o e e I_{t1} I_{t11} I_{t12} nd I_{t2} I_{t21} I_{t22} e e fo $a \in$, /) nd $\eta >$

$$I_{t11} = E\{(Y_t - \nu_p)I(\nu_p \le Y_t < \nu_p)I(\nu_p \ge \nu_p + n^{-a}\eta)\},$$
$$I_{t12} = E\{(Y_t - \nu_p)I(\nu_p \le Y_t < \nu_p)I(\nu_p < \nu_p < \nu_p + n^{-a}\eta)\},$$
$$I_{t21} = E\{(Y_t - \nu_p)I(\nu_p > Y_t \ge \nu_p)I(\nu_p \le \nu_p - n^{-a}\eta)\} \quad \text{nd}$$
$$I_{t22} = E\{(Y_t - \nu_p)I(\nu_p > Y_t \ge \nu_p)I(\nu_p > \nu_p > \nu_p - n^{-a}\eta)\}.$$

App y n e C y z neq y fo k nd

$$|I_{tk1}| \le \sqrt{E(\nu_p - \nu_p)^2 P(|\nu_p - \nu_p| \ge n^{-a}\eta)}$$

en Le nd e f c $E\ \nu_p - \nu_p)^2$ $O\ n^{-1})$ p y

$$I_{tk1} \rightarrow \quad \text{e ponen y f} \qquad \text{A}$$

o e e I_{t12} e no e $|I_{t12}| \leq E\{ Y_t - \nu_p) I\ \nu_p \leq Y_t < \nu_p \quad n^{-a}\eta)\}$. e n

$$I_{t12} \leq \int_{\nu_p}^{\nu_p + n^{-a}\eta} dy\ z - \nu_p) f\ z) dz \quad O\ n^{-2a})$$

n e e c y e pp o e c n o I_{t22} $O\ n^{-2a})$ e e nd A e n y oo n a $-$ / γ e e $\gamma >$ y

$$E\ W_t) \quad o\ n^{-1+\kappa}) \qquad \text{A}$$

fo n y po e κ n n p e

$$E\Big[n^{-1} \sum Y_t - \nu_p)\{ I\ Y_t \geq \nu_p) - I\ Y_t \geq \nu_p) \} \Big] \quad o\ n^{-1+\kappa}) \qquad \text{A}$$

e no con de $Var\ W_i)$ o $a \in$, /)

$$\begin{aligned}
E\ W_t^2) \quad & E\Big[Y_t - \nu_p)^2 \{ I\ Y_t \geq \nu_p) - \ I\ Y_t \geq \nu_p) I\ Y_t \geq \nu_p) \quad I\ Y_t \geq \nu_p) \} \Big] \\
& E\Big[Y_t - \nu_p)^2 \{ I\ \nu_p > Y_t \geq \nu_p) \quad I\ \nu_p > Y_t \geq \nu_p) \} \Big] \\
& E\Big[Y_t - \nu_p)^2 I\ \nu_p \leq Y_t < \nu_p) \{ I\ \nu_p \geq \nu_p - n^{-a}\eta) \quad I\ \nu_p < \nu_{pn^{-a}}\eta \ \ a_{\eta})\} \Big] \\
& E\Big[Y_t - \nu_p)^2 I\ \nu_p > Y_t \geq \nu_p) \{ I\ \nu_p \geq \nu_p \quad n^{-a}\eta) \quad I\ \nu_p < \nu_p \quad n^{-a}\eta) \} \Big].
\end{aligned}$$

No e

$$\begin{aligned}
& E\{ I\ \nu_p \leq Y_t < \nu_p) I\ \nu_p \leq \nu_p - n^{-a}\eta) \} \leq P\ |\nu_p - \nu_p| \geq n^{-a}\eta) \quad \text{nd} \\
& E\{ I\ \nu_p > Y_t \geq \nu_p) I\ \nu_p > \nu_p \quad n^{-a}\eta) \} \leq P\ |\nu_p - \nu_p| \geq n^{-a}\eta)
\end{aligned}$$

con e e ze o e ponen y f p ed y Le App y n e C y z neq y e

$$\begin{aligned}
& E\{ Y_t - \nu_p)^2 I\ \nu_p \leq Y_t < \nu_p) I\ \nu_p \leq \nu_p - n^{-a}\eta) \} \quad \text{nd} \\
& E\{ Y_t - \nu_p)^2 I\ \nu_p > Y_t \geq \nu_p) I\ \nu_p \geq \nu_p \quad n^{-a}\eta) \}
\end{aligned}$$

con e e q ze o e ponen y f e en pp y n e e od e A e

$$E\{ Y_t - \nu_p)^2 I\ \nu_p \le Y_t < \nu_p) I\ \nu_p \ge \nu_p - n^{-a}\eta)\} \quad O\ n^{-3a}) \quad \text{nd}$$
$$E\{ Y_t - \nu_p)^2 I\ \nu_p > Y_t \ge \nu_p) I\ \nu_p < \nu_p \quad n^{-a}\eta)\} \quad O\ n^{-3a})$$

n y e $E\ W_t^2)$ $o\ n^{-3/2+\kappa})$ nd A e n $Var\ W_t)$ $o\ n^{-3/2+\kappa})$ By y od fy n e o e de on fo $Var\ W_t)$ y e o n fo ny t_1, t_2 $Cov\ W_{t_1}, W_{t_2})$ $o\ n^{-3/2+\kappa})$ e efo e

$$Var\left[n^{-1} \sum_{i=1}^{n} Y_t - \nu_p)\{I\ Y_t \ge \mu_p) - I\ Y_t \ge \nu_p)\} \right] \quad o\ n^{-3/2+\kappa}) \qquad \text{A}$$

q e A e d y e e e e

LEMMA Le β $np)^{-1} \sum Y_t G_h\ \nu_p - Y_t)$ nd η $n\ )^{-1} \sum_{i=1}^{n} Y_t K_h\ \nu_p - Y_t)$ nde e cond on

$$Cov\left[\beta, \{p - S_h\ \nu_p)\}\{f\ \nu_p) - f\ \nu_p)\}\right] \quad o\ n^{-1}$$
$$Cov\left[\beta,\ \eta - \eta)\{p - S_h\ \nu_p)\}\right] \quad o\ n^{-1}$$
$$\text{c)}\ Cov\left[\{p - S_h\ \nu_p)\},\ \eta - \eta)\{p - S_h\ \nu_p)\}\right] \quad o\ n^{-1}$$

OO e on y p e n e p oof of e p oof fo e e e De ne β $E\ \beta)$ Le $\beta - \beta$ $n^{-1} \sum \psi_1\ Y_t)\ f\ \nu_p) - f\ \nu_p)$ $n^{-1} \sum \psi_2\ Y_t)$ $O\ ^2)$ nd $p - F_h\ \nu_p)$ $n^{-1} \sum \psi_3\ Y_t)$ $O\ ^2)$ fo e f nc on ψ_j j , nd $E\{\psi_j\ Y_t)\}$ o n nce $\psi_2\ Y_t)$ $K_h\ \nu_p - Y_t) - E\{K_h\ \nu_p - Y_t)\}$ nd $\psi_3\ Y_t)$ $G_h\ \nu_p - Y_t) - E\{G_h\ \nu_p - Y_t)\}$ n e pp o n B n ey p

$$A \quad |E\left[\ \beta - \beta)\{p - S_h\ \nu_p)\}\{f\ \nu_p) - f\ \nu_p)\}\right]$$
$$\le \quad n^{-2} \sum_{i \ge 1, j \ge 1, i+j \le n} |E\{\psi_1\ Y_1)\psi_2\ Y_i)\psi_3\ Y_{i+j})\}| \qquad O\ n^{-1}\ ^4 \quad n^{-2}\ ^2) \qquad \text{A}$$

e e nd c e e d e en pe on on e ee nd ce Le p δ q δ nd s^{-1} $-p^{-1} - q^{-1}$ fo e po e δ e D ydo neq y

$$|E\{\psi_1\ Y_1)\psi_2\ Y_i)\psi_3\ Y_{i+j})\}| \le \quad ||\psi\ Z_1)||_p ||\psi_2\ Y_t)\psi_3\ Z_{i+j})||_q \alpha^{1/s}$$

e e o e p oy n e o

By oo n r n^a fo $a \in$, nd k' n^c $c \in$, $-a$ e c n o e ef nd de of A con e e o $n \to \infty$ ence

$$_n \xrightarrow{p} \quad n \to \infty. \qquad \text{A}$$

e efo e $S_{n,1}$ $n^{-1/2} \sum_{j=1}^{r} W_{j,n}$ o_p

By pp y n e neq y e ed n Yo oy nd e con c on of $W_{j,n}$

Le η $E\{p^{-1}Y_tK_h\ Y_t - \nu_p\}$ $p^{-1}\int\ \nu_p -\ \ K\ \ f\ \nu_p -\ \ d$ $p^{-1}\nu_p f\ \nu_p$ $O\ ^2$

n nd d de on fo α n q ence fo n nce o en n Bo

e $Cov\{\ np^{-1}\sum Y_tK_h\ \nu_p - Y_t\ \nu_{p,h} - \nu_p\}$ $O\ n^{-1}$ ence f A

$$E\{\ np^{-1}\sum Y_tK\ \nu_p - Y_t\ \nu_{p,h} - \nu_p\} \quad \eta E\ \nu_{p,h} - \nu_p \quad O\ n^{-1}$$

$$-\tfrac{1}{2}p^{-1}\nu_p f'\ \nu_p\ ^2\sigma_K^2 \quad o\ ^3 \quad O\ n^{-1} \quad \text{A}$$

Co ne A A nd A

$$E\ \mu_{p,h} \quad \mu_p + \tfrac{1}{2}p^{-1}\sigma_K^2\ ^2 f\ \nu_p \quad o\ ^2 \quad O\ n^{-1}$$

e e e en n

e no de e e nce of $\mu_{p,h}$ Le A_1 $np^{-1}\sum_{t=1}^{n}\{Y_tG_h\ \nu_p - Y_t - Y_tK_h\ \nu_p -$ $Y_t\ \nu_{p,h} - \nu_p\}$ e e e d n o de e of e e p n on A

en

$$Var\ A_1 \quad Var\{\ np^{-1}\sum Y_tG_h\ \nu_p - Y_t\} \quad Var\{\eta\ \nu_{p,h} - \nu_p\}$$

$$- \quad Cov\{\ np^{-1}\sum Y_tG_h\ \nu_p - Y_t\ \eta\ \nu_{p,h} - \nu_p\}. \qquad \text{A}$$

e Q e

$$Var\{\ np^{-1}\sum Y_tG_h\ \nu_p - Y_t\}$$

$$n^{-1}p^{-2}\Big[Var\{Y_tG_h\ \nu_p - Y_t\} \quad \sum_{k=1}^{n-1} \quad - k/n\ Cov\{Y_1G_h\ \nu_p - Y_1\ Y_{k+1}G_h\ \nu_p - Y_{k+1}\}\Big].$$

Le c_K $\int_{-\infty}^{\infty} K\ \ d\ \int_{-\infty}^{u} K\ \ d$ y e o n

$$Var\{Y_tG_h\ \nu_p - Y_t\} \quad \int z^2G_h^2\ \nu_p - z\ f\ z\ dz - p^2\mu_p^2 \quad O\ ^2$$

$$\int_{-\infty}^{\infty} K\ \ d\Big[\int_{-\infty}^{u} K\ \ d\ \{\int_{\nu_p}^{\infty} z^2 f\ z\ dz \quad \int_{\nu_p - hv}^{\nu_p} z^2 f\ z\ dz\}$$

$$\int_{u}^{\infty} K\ \ d\ \{\int_{\nu_p}^{\infty} z^2 f\ z\ dz \quad \int_{\nu_p - hu}^{\nu_p} z^2 f\ z\ dz\} - p^2\mu_p^2 \quad O\ ^2$$

$$Var\{Y_tI\ Y_t \geq \nu_p\} - \quad \nu_p^2 f\ \nu_p\ c_K \quad O\ ^2 \qquad \text{A}$$

on A nd Le e n

$$Var\{\ np^{-1}\sum Y_tG_h\ \nu_p - Y_t\} \quad p^{-2}Var\{\phi_1\ \nu_p\} - \ n^{-1}\ \nu_p^2 f\ \nu_p\ c_K \quad o\ n^{-1} \qquad \text{A}$$

e cond e on e nd de of A

$$Var\{\eta\ \nu_{p,h} - \nu_p\} \quad \eta - \eta\ \nu_{p,h} - \nu_p\}$$

$$\eta^2 Var\ \nu_{p,h} \quad \eta Cov\ \nu_{p,h},\ \eta - \eta\ \nu_{p,h} - \nu_p\} \quad Var\{\ \eta - \eta\ \nu_{p,h} - \nu_p\}.$$

y e o n y n e f c $\eta \quad p^{-1}\nu_p f\ \nu_p \quad O \quad ^2$

$$\eta^2 Var\ \nu_{p,h} \quad p^{-2}\nu_p^2 Var\{n^{-1}\sum_{t=1}^{n} I\ Y_t > \nu_p\} - \ p^{-2}n^{-1}b\nu_p^2 f\ \nu_p c_K \quad o\ n^{-1}b \qquad \text{A}$$

e neq y en n Yo oy fo α n q ence

$$E\ \nu_{p,h} - \nu_p^4 \le Cn^{-2} \quad \text{nd} \quad E\ \eta - \eta^4 \quad O\ n^{-2}\ ^{-3}$$

App y n e C y z neq y nd Le

$$Var\{\ \eta - \eta\ \nu_{p,h} - \nu_p\} \quad O\ n^{-2}\ ^{-3/2} \quad o\ n^{-1} \quad \text{nd} \qquad \text{A}$$

$$Cov\{\eta\ \nu_{p,h} - \nu_p\ p^{-1}\ \eta - \eta\ \nu_{p,h} - \nu_p\} \quad o\ n^{-1} \qquad \text{A}$$

C ne A A nd A

$$Var\{\eta\ \nu_{p,h} - \nu_p\} \quad p^{-2}\nu_p^2 Var\{\phi_2\ \nu_p\} - \ p^{-2}n^{-1}\ \nu_p^2 f\ \nu_p c_K \quad o\ n^{-1} \qquad \text{A}$$

Le e co nce e on e nd de of A

$$Cov\{\ np^{-1}\sum_{t=1}^{n} Y_t G_h\ \nu_p - Y_t\ \eta\ \nu_{p,h} - \nu_p\}$$

$$Cov\{\ np^{-1}\sum_{t=1}^{n} Y_t G_h\ \nu_p - Y_t\ \eta f^{-1}\ \nu_p n^{-1}\sum_{t=1}^{n} G_h\ \nu_p - Y_t\} \quad o\ n^{-1}$$

$$np^2{}^{-1}\nu_p \Big[Cov\{Y_t G_h\ \nu_p - Y_t\ G_h\ \nu_p - Y_t\}$$

$$\sum_{k=1}^{n-1} \quad - k/n\ Cov\{Y_1 G_h\ \nu_p - Y_1\ G_h\ \nu_p - Y_{k+1}\} \Big] \quad o\ n^{-1}$$

nce $Cov\{Y_t G_h\ \nu_p - Y_t\ G_h\ \nu_p - Y_t\} \quad p \quad - p\ \nu_p - \ \nu_p f\ \nu_p\ c_K \quad o$

$$Cov\{\ np^{-1}\sum_{t=1}^{n} Y_t G_h\ \nu_p - Y_t\ \ np^{-1}\sum_{i=1}^{n} Y_t K_h\ \nu_p - Y_t\ \nu_{p,h} - \nu_p\} \qquad \text{A}$$

$$n^{-1}p^{-2}\nu_p Cov\{\phi_1\ \nu_p\ \phi_2\ \nu_p\} - \ n^{-1}p^{-2}\nu_p^2 f\ \nu_p\ c_K \quad o$$

e A 4 A 4 nd A 4 o A 4 e no e e cond o de e of $O(n^{-1})$ c nce o e e nd e efo e

$$Var(\mu_p) \quad p^{-1}n^{-1}\sigma_0^2(p,n) \quad o(n^{-1}) \qquad \text{A 4}$$

e 4

e p o c n B y of $\nu_{p,h}$ c n e e ed f A 4 y n e e oc n od n e p oof of eo

NC

A zne De en e M nd e D 4 C e en e e of *M the tic Fin nce* **9**

Be n nd nc 4 De ec n nd ode n dependence M n p nd p pe p e n ed n n e M od n nce Confe ence C n r ece e c d e d e ne Be n 4 ey nd McNe A 4 nd e pec ed o f n po fo o of dependen c ed concep nd p c c n *ourn of B nking nd Fin nce*

enon C o e e nd C L edo 4 c o y Mode dden M o Mode nd c App c on *Bernoulli*

o e o C c e O nd L en 4 en y n y of e *ourn of e piric n nce*

o on 4 *t Risk* nd on Ne Yo Mc

Le d e e M L nd en nd oo zen 4 *E tre es nd Re ted Properties of R ndo Sequences nd Processes* p n e Be n

M y nd e D 4 Nonp e c e on nd den c on of non ne e e *Econo etric Theory* **11**

e D nd M 4 *St tistic An ysis of E tre e es ith pp c tions to insur nce n nce hydro ogy nd other e ds* nd ed Bo on B e

o n M 4 A cen eo e nd e α n cond on *Proc N t Ac d Sc S A* **42**

c e O 4 Nonp e c e on of cond on e pec ed o f en e e eo

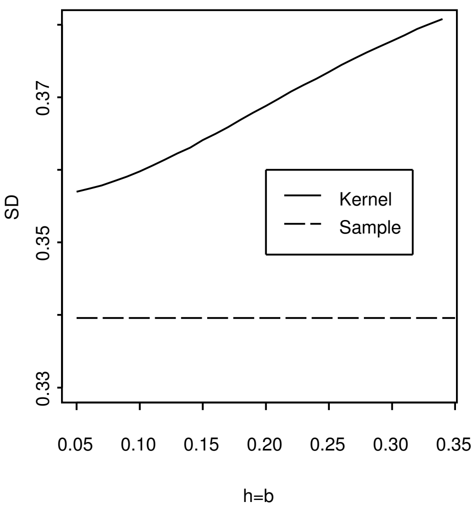
c e O 4 Nonp e c e on nd n y n y of e pec ed o f *M the tic Fin nce* **14**

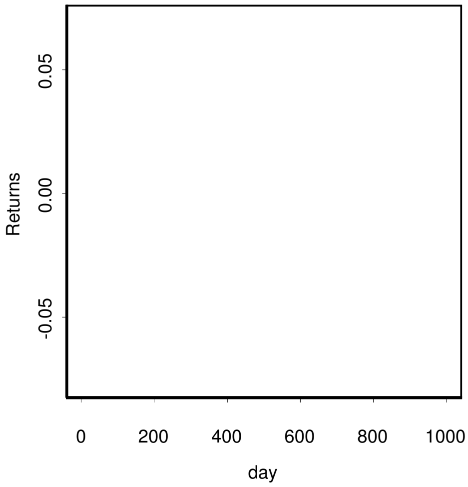
e e nd M on 4 e ne q n e e q *ourn of A eric n St tistic Associ tion* **5**

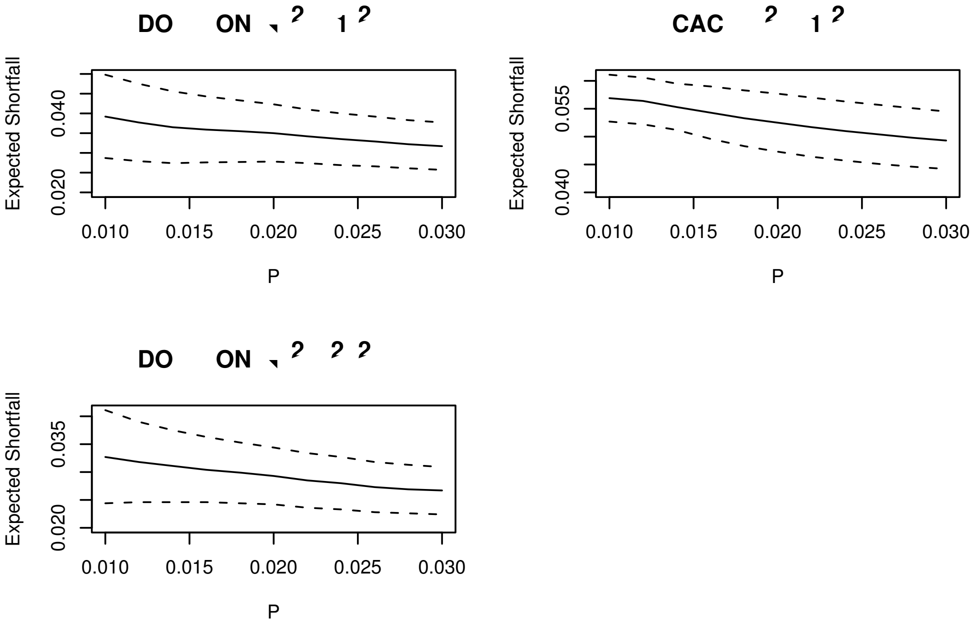
Yo oy 4 Mo en o nd fo on y n q ence *Pro ity Theory nd Re ted Fie ds* **52**

Y 4 e B d ep e n on of p e q n e fo q ence of on y n ndo e *St tistics nd Pro ity Letters* **24**

(a) ES Estimates




0.05
0.00
-0.05
Returns
0
200
400
600
800
1000
day


DO ON 1
CAC 1
DO ON
Expected Shortfall
0.020
0.040
0.055
0.035
0.010
0.015
0.020
0.025
0.030
P

T … Esti … tes for $\nu_{0.01}$ … $\mu_{0.01}$ … *nd St … nd … rd Errors … S.E*

	CAC			Do … one		
Ye	$\nu_{0.01}$	μ_p		$\nu_{0.01}$	μ_p	