Public-Private Mix of Health Expenditure: A Political Economy and Quantitative Analysis*

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1 Introduction

Achieving a good health status in the overall population is one of the most important goals in every society, not only because it enhances people's life expectancy by reducing both the mortality and morbidity rates, but also because it can improve workers' labor productivity and hence allow a society to consume more output. There are various factors that can improve health status and life expectancy. Among those most important factors are public health care and private health care. The former includes public health policies or programs that provide public hospitals, immunization, disease control and diagnostic health screening, invest in new medical facilities, and promote healthy environment through, e.g., reducing air and water pollutions. The latter refers to private decisions on eating healthy food, taking preventive medicines and vitamins, and having preventive and diagnostic health screenings.

Although the total spending on public and private health care has been rising, and consequently health status has been improving in most countries, there are considerable variations in the mixture of public and private health spending over time and across countries. For instance, the share of public health in total health expenditure has increased by more than 10 percent since the 1970s in the U.S., Austria, Greece, and Japan, while it has decreased by more than 10 percent in the Czech Republic, Norway and the UK. Across OECD countries, the share of public health ranges from less than 50 percent (e.g., the U.S.) to more than 80 percent (e.g., Denmark, Norway, Sweden, Japan, the UK, the Czech Republic) in the 2000s.

This paper aims to shed light on the interaction between public and private financing of health care in a society. Understanding this relationship can help policy makers to allocate resources to health care more efficiently so as to achieve better health outcomes. This is particularly relevant for discussions among policy makers as well as economists regarding the role of private and public sectors in providing health care and their implications for efficiency and equity of the society.

In particular, we study how the public-private mix of health expenditure is chosen by people in a democratic society collectively, when people can choose between public and private health care in improving their life expectancies. We construct an overlapping generations model to explore how public and private health spending are determined by utility-maximizing agents with heterogenous income through majority voting, and how their decisions are shaped by the degree of substitutability between public and private health, income distribution and other economic factors, as well as preferences. Furthermore, the model is calibrated to conduct a quantitative exercise to investigate how well the model can explain the observed differences in the mixture of health expenditure across a group of advanced democratic countries.

In the model, agents live for three periods: childhood, young adulthood and old adulthood. In young adulthood, agents receive exogenous heterogeneous income, and they decide how much to consume, to save for old adulthood, and to invest in private health care, and they vote for the income tax to be used to finance public health. Agents' survival probabilities to old adulthood are endogenously determined by a CES composite of the public and private health expenditure. For the two special cases that public and private health are complements or perfect substitutes, we show that the voting equilibrium is unique and locally stable, and derive the closed form solution for the steady state majority choice of tax rate. For the general case, we are able to show the existence of a voting equilibrium and derive the equations that implicitly determine the equilibrium majority choice of tax rate. Instructed by the equilibrium equations, we then calibrate the model and conduct a quantitative analysis.

The baseline values for parameters are calibrated to match moments of the Canadian data, including several important moments that capture the life expectancy, relative size and composition of health expenditure in Canada. The comparative static results suggest that the size of public health spending relative to national income and the share of public health in total health expenditure are quite sensitive to the degree of substitutability between private and public health and the expenditure weight in the CES function that indicate the relative importance of public and private health. We further infer these two parameters for each country using country-speci…c data, and construct the model predicted shares of public health in total health expenditure for each country in the sample. The results show that the predicted mixture of health expenditure matches the data quite well for the majority of countries, with an overall correlation of 0.44 between predicted shares of public health and corresponding data values. We then discuss several factors the model abstracts from that may have important implications for the public-private mix of health expenditure, such as demographic structure, pricing of health care services, and the composition of funds within public and private …nancing of health care.

The contributions of this paper are two-fold. The …rst contribution is a theoretical one. It is one of the few studies that aims to explain the coexistence of private and public health care. Epple and Romano (1996) and Gouveia (1997) are among the …rst to address this issue. Following the strand of literature on the socialization of commodities, they focus on the public provision of a private good –health care– through majority voting in a static 7(t)8(h)1ies6n thads6n10 existence of public health insurance system in the absence of complete markets to insure (such as the U.S. and the UK) receive some empirical support from country-specific studies. Finally, there is a relatively close match between the predicted shares of public health in total health spending and the corresponding data values for the group of 22 advanced democratic countries. These results suggest that our model provides a promising framework to study the determination of public and private health spending. The quantitative results also provide important insights for empirical work on health. The model has identified several important

(2) is a reduced-form representation of the structure of public and private health care in a society. It allows for general substitutability between these two in forming the health capital of the society.

The assumption above implies that an agent's mortality later in life is determined by her health status in childhood, which depends on her parents' choices of health expenditure. Empirical evidence on the importance of health status in childhood for mortality later in life is well documented in Van Den Berg, Lindeboom and Portrait (2006) and Ferrie and Rolf (2011). There is also plenty of empirical evidence that poor childhood health conditions cause higher morbidity in later life (see, e.g., Kuh and Ben-Shlomo, 1997, and Blackwell, Hayward, and Crimmins, 2001). For instance, Blackwell et al. (2001) find a strong link between poor childhood health and a range of chronic diseases, such as cancer, lung disease, cardiovascular conditions and arthritis. Such chronic diseases are increasingly contributing to the total deaths and significant growth in medical expenditures across the world.[2] Similar assumptionsoimc56boiin(e)9(r)-394(e)1iiH(e)1togheoghturpt11(i)6(c)9(a)11(e)-350(su(o)11(n)11(d)11 and thus impose great difficulty in the modelling and calibration, would obscure our main focus.[3]

The lifetime utility of agent i at time t is defined over her consumption in young adulthood, $c_{i,t} \in R_+$, consumption in old adulthood, $d_{i,t+1} \in R_+$, and health capital, $\hat{H}_{i,t} \in R_+$:[4]

$$U_{it}$$

Given a tax rate τ_t, agent i's utility maximization problem is to choose $s_{i,t}$ and $h_{i,t}$ to maximize (3) subject to (4) and (5).

Public health expenditure, H_t, is financed by income taxes collected from young adults in period t. Government budgets are balanced in every period:

$$H_t = \tau_t y_t. \tag{6}$$

The tax rates prevailing in each period are endogenously determined by a majority voting mechanism. That is, in period t, each young adult votes on her preferred tax rate, which would be the tax rate that maximizes her indirect utility. The collective choice of the tax rate, τ_t, is determined by the majority rule where the decisive voter is the individual with the median income.

It is not easy to characterize the voting equilibrium analytically for the general form of health capital in (2). Therefore we first consider two special cases: $\rho = 0$ and $\rho = 1$. For each case, we are able to show the existence, uniqueness, and stability of the equilibrium tax rate under majority voting, as well as some analytical results regarding how the equilibrium tax rate (ratio of public health expenditure to mean income) and the public-private mix of health expenditure depend on important economic variables. We then provide a quantitative exercise for the general case.

3 Special Cases

3.1 Public and Private Health are Complements

We first consider the special case that $\rho = 0$ in (2), then the health capital takes the Cobb-Douglas form:

$$\hat{H}_{i,t-1} = H_{i,t-1}^{\theta_H} h_{i,t-1}^{\theta_h}. \tag{7}$$

Under this form, public and private health care are substitutable, but have a low elasticity of substitution 1, or in other words, they are more complementary or supplementary in forming the society's health capital. In this sense, we refer to this case as public and private health being complements in Cobb-Douglas form. In practice, public and private health care are complementary in many ways. For instance, public health expenditure that provides public vaccination, such as ‡u shot, complements private health expenditure on nutritious food in reducing individuals' risk of getting communicable diseases.

Solving the utility maximization problem formulated earlier yields agent i's optimal private saving and private health spending:

$$s_{it} = \frac{p_{i;t}}{(1 + \quad p_{i;t})}\left((1 - \quad {}_{t})y_{i;t} - h_{i;t}\right), \tag{8}$$

$$h_{i;t} = \frac{{}_{h}}{1 + \quad p_{i;t} + \quad {}_{h}}(1 - \quad {}_{t})y_{i;t}. \tag{9}$$

Eq. (8) implies that an increase in the probability of survival increases private savings as young adults who expect to live longer are e¤ectively more patient and more willing to save for old adulthood consumption. It is also clear from (9) that private health expenditure is a normal good. What is relatively new here is that an increase in an agent's survival probability reduces her private health investment. This occurs because the agent's own adulthood consumption competes with the private health investment. The negative relationship between life expectancy and private health spending also holds for the special case of public and private health being perfect substitutes (see Section 3.2) and the general substitutability case (see Section 4). This prediction of the model receives empirical support from the data we use for the quantitative exercise.[5]

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[5] See Section 4 for more details of the data sample. The following facts documented from the data are consistent with the model's prediction. First, the correlation between life expectancy at birth and private health spending is negative for all 22 countries in our sample. Second, the correlation between life expectancy at birth and the share of private health in total health spending is negative for 15 out of the 22 countries. In particular, for Canada, the country used for the baseline calibration, these two correlations are -0.7 and -0.45, respectively.

To derive the equilibrium tax rate preferred by a majority of voters, we first find the indirect utility of agent i by substituting Eq. (8) and (9) into the utility function (3):

$$V_{i;t} = (1 + \; p_{i;t} + \; _{h}) \ln (1 - \; _{t}) + \; _{H} \ln (\; _{t}) + \; _{c}, \tag{10}$$

where $_{c}$

and

$$\hat{H}_{m;t-1} = \quad _{m;t-1} \quad _{m;t-1}, \tag{13}$$

where $_{m;t-1} = [(\ _h = \ _H)(y_{m;t-1}=y_{t-1})]\ ^h y_{t-1}$. Note that in this special case the ratio of median private health to public health expenditure is independent of the equilibrium tax rate. In fact, it is the product of two ratios: the share of private relative to public health expenditure in the composite health service, and the ratio of median income to mean income, which is an important measure of income inequality. It is obvious from (12) that the median voter prefers to have higher private health relative to public health when income inequality is lower ($y_{m;t}=y_t$ is higher), and when the share of private relative to public health expenditure is higher.

Note that Eq. (11) characterizes the majority choice of tax rate in a recursive manner, that is, $_{m;t}$ depends on the majority choice of tax rate in the previous period, $_{m;t-1}$. An increase in $_{m;t-1}$ reduces $_{m;t}$ because an increase in $_{m;t-1}$ increases the survival probability, $p_{m;t} = p(\hat{H}_{m;t-1})$, via its positive effect on the health capital, $\hat{H}_{m;t-1}$ in (13). That is, the first derivative of $_{m;t}$ with respect to $_{m;t-1}$ is negative. It can also be shown that the second derivative of $_{m;t}$ with respect to $_{m;t-1}$ is positive. Hence Eq. (11) implies a unique steady state of the majority choice of tax rate, if we consider an economy with a time-invariant distribution of income, $F_t(\cdot) = F(\cdot)$ for all t.

To further characterize the steady state equilibrium, we consider the following parametric form for the survival probability $p_{i;t}$:[8]

$$p_{i;t} = p\left(\hat{H}_{i;t-1}\right) = \frac{\hat{H}_{i;t-1}}{1 + \quad \hat{H}_{i;t-1}}, \tag{14}$$

where $\ > 1$. This form implies that the survival probability is strictly increasing and strictly

[8] A similar functional form for the probability of survival is assumed in Chakraborty (2004).

concave in health capital. Then Eq. (11) can be written as:

$$_{m;t} =$$

leads to lower private savings for old adulthood and hence more income in young adulthood can be allocated to private health investment.

Substituting the expression for private savings in (18) into the utility function, we obtain the indirect utility for agent i:

$$V_{i,t} = (1 + \quad p_{i,t}) \ln((1 - \quad_t)y_{i,t} - h_{i,t}) + \quad \ln(\quad_H H_t + \quad_h h_{i,t}) + \quad_{PS},$$

where $_{PS} = -(1 + \quad p_{i,t}) \ln(1 + \quad p_{i,t}) + \quad p_{i,t} \ln(\quad (1 + r_{t+1}) p_{i,t}/p_t)$, which is independent of the current tax rate, $_t$. It can be shown that $\partial^2 V_{i,t}/\partial \quad_t^2 < 0$ (regardless of whether $h_{i,t}$ is zero or positive) such that voters' preferences are single-peaked in $_t$. Then the equilibrium tax rate under majority voting, $_{m,t}$, is the tax rate that maximizes the median voter's indirect utility.

If the following condition holds (obtained by rearranging (20)),

$$_{m,t} \leq \frac{}{\quad + (1 + \quad p_{m,t})\frac{H}{h}\frac{y}{y_m}} \equiv \hat{}_{m,t}, \tag{21}$$

$h_{m,t}$ is given by the second expression in the bracket in (19), so we obtain the following expression for $\partial V_{m,t}/\partial \quad_{m,t}$:

$$\frac{\partial V_{m,t}}{\partial \quad_{m,t}} = (1 + \quad p_{m,t} + \quad) \frac{y_{m,t}\left(\frac{H}{h}\frac{y_t}{y_{m,t}} - 1\right)}{(1 - \quad_{m,t})y_{m,t} + \frac{H}{h}H_t}.$$

Assume that the ratio of median to mean income is constant over time, i.e., $y_{m,t}/y_t = y_m/y$, and define $F \equiv (\quad_H/\quad_h)(y/y_m)$. If $F = 1$, $\partial V_{m,t}/\partial \quad_{m,t} = 0$ for any $_{m,t} \in [0, \hat{}_{m,t}]$ such that the equilibrium tax rate is not unique. Hence, we focus on the case that $F \neq 1$.

First consider F

relatively low (low y/y_m) or public health is relatively less effective (low ${}_H/{}_h$), or both. Hence a majority of voters tend to vote for a low level of public health, which serves as a redistribution device in the model, and replace public health with private health as they are perfect substitutes in improving longevity.

Next we consider $F > 1$, then $\partial V_{m;t}/\partial \; {}_{m;t} > 0$ for all t, so the median voter prefers a the following properties: (i) $_m \in (0;1)$; (ii) $_m$ is locally stable and converged with damped oscillations; (iii) $_m$ rises with a rise in y_m=y (keeping y constant); (iv) $_m$ falls with a rise in y (keeping y_m=y constant).

The comparative static property stated in (iii) is di¤erent to that found in the complement case and in most of the existing literature. It states that the majority choice of tax rate increases with a reduction in the degree of income inequality. This may sound puzzling. However, note that when there is a reduction in income inequality (a fall in y=y_m), for $F > 1$ to hold, $_H = _h$ needs to be relatively large, implying that public health is much more e¤ective relative to private health. Therefore a majority of individuals tend to vote for a higher tax rate to substitute public health for private health .

4 General Case: A Quantitative Exercise

Now consider the general form of health capit of h5Td[9m del417(r)11(e)9(d)2.823Td[()]322.2997T

expenditure (h_m=H), as characterized in Eq. (25), decreases with an increase in income inequality (lower y_m=y) and with an increase in $_H$, which indicates the importance of public health relative to private health, and decreases (increases) with an increase in the elasticity of substitution between private and public health 1=(1 −) if *F* is greater than one (less than one). However, it is di¢ cult to derive further analytical results. Instead, we calibrate the parameters and conduct a quantitative analysis for a sample of advanced democratic countries.

4.1 Baseline Calibration

Parameters of the model include: the discount factor ( ), the interest rate (r), the utility weight attached to the health capital ( ), the parameter in the parametric form of survival probability ( ), the expenditure weights in the CES function of health capital ($_H$, $_h$ ≡ 1 − $_H$), and the parameter measuring the degree of substitution between public and private health ( ). We calibrate these parameters to match certain characteristics of the Canadian data (2000-2009). Canada is chosen for its well-established universal public health care system as well as data availability.

First, we set a period in the model to be 30 years, that is, childhood is from 0 to 30 years old, young adulthood is from 31 to 60 years old, and old adulthood is from 61 to 90 years old. Then and r can be set to match the average annual real interest rate in Canada, which is around 2:51% according to World Development Indicator-2011. That is, $r = (1 + 0{:}0251)^{30} - 1$, and = 1=(1 + r). To calibrate , note that the parametric form given in (14) implies that the maximum survival probability is given by 1= , which would be achieved when the health capital approaches in…nity. So we set to match a maximum survival probability of 0:9, i.e., = 1=0:9.[9]

The parameters , and $_H$ are not directly deducible from the data. We calibrate them jointly using the steady state versions of the three equations that characterize the solution

[9] The sensitivity analysis below shows that the equilibrium tax rate and the public-private mix of health expenditure are not sensitive to .

income inequality measure that is taken as exogenous, y_m/y, is equal to 0.8659 according to OECD.Stat Extracts - 2011. Given the moments above, the ratio of per capita public health to median private health expenditure, H/h_m, appearing in Eq. (25) to (27), is obtained as $H/h_m = 1/\left[\left(h_m/h\right)\left(h/H\right)\right]$.[14]

The calibration procedure is as follows. First, can be determined independently of and $_H$, by combining Eq. (25) and (26):

$$= \frac{(1 + \ p_m)\left(1 + \frac{H}{h_m}\frac{y_m}{y}\right)}{\frac{1-\ _m}{_m}\frac{y_m}{y}\frac{H}{h_m} - 1}. \tag{28}$$

For a given H, and $_H$ are solved jointly from Eq. (25) and (27). We assume that the distribution of income is a log-normal distribution with parameters and , and calibrate and to match the scaled mean income and the degree of inequality in the Canadian data, that is, $= \ln(y \cdot (y_m/y))$, where y is the scaled mean income given by $H/\ _m$, and $= \sqrt{2\ln(y/y_m)}$. Then we draw 20,000 income realizations from this distribution, and for each income draw, $y_{i,t}$, we solve the corresponding private health expenditure, $h_{i,t}$, from (24).[15] Then h is calculated as the mean of $h_{i,t}$'s, and a value for the ratio of private health to public health h/H is obtained. If this value is different from the value of h/H in the data, another H is chosen and the process described above is repeated, until the computed value of h/H matches its data counterpart.

Table 1 summarizes the calibrated parameter values and the data moments used for calibration. Recall that is an agent's utility weight assigned to the health capital that would determine her offspring's survival probability, so it indicates the degree of altruism towards

Table 1: Baseline Cabibration

Parameters	Description	Calibrated Value
	Discount factor	1=(1 + 2:51%)
	Parameter in the survival probability	1=0:9
	Agent's utility weight assigned to health capital	0:1386
	Degree of substitution between public and private health	0:6162
$_H$	Expenditure weight of public health in health production	0:58
H	Public health	9:15
Variables	**Description**	**Data Value**
r	Interest rate	2:51%
p	Maximum survival probability	0:9
$p_{m;t}$	Median survival probability	0:784
$_m \equiv H=y$	Ratio of public health to national income	6:83%
$H=(H + h)$	Share of public health in total health expenditure	70:14%
$h_m=h$	Ratio of median to mean private health expenditure	0:6962
$y_m=y$	Ratio of median to mean income	0:8659

offspring. The calibrated value of is about 40 percent of the median effective discount factor p_m. This value is slightly lower than the values used in most of the quantitative studies that consider altruism toward children's education or future earnings.[16] The value of implies an elasticity of substitution between public and private health of 2:6. The value of $_H$ is a bit higher than 0:5, suggesting that public health plays a slightly larger role than private health in the society's health capital. Due to the lack of relevant empirical or quantitative studies, we cannot compare the calibrated values for and $_H$ with other studies. Nevertheless, these values seem realistic for the Canadian economy.

Under the baseline calibration, the numerical derivatives of public health (H), mean private health (h), and total health (H + h) with respect to average income (y) can be calculated, and hence the income elasticities of total health, public health, and private health expenditure are computed as 1:0074, 0:9692 and 1:0901, respectively. The income elasticity of health expenditure has been an important subject addressed in the empirical literature.

[16] Most of the literature that considers altruism assume parents care about their children's education or children's future income rather than children's health status in a life-cycle model. In their numerical exercises, most studies assume a relatively high degree of parental altruism (e.g., 0.65 in Osang and Sarkar (2008) and Raut (2003), and 0.5 in Kalemli-Ozcan (2002)), and a few studies assume a relatively low degree of altruism (e.g., 0.3 in Tang and Zhang (2007) and 0.1 in Pecchenino and Utendorf (1999)).

The values of the income elasticity of total health expenditure produced by the model is in line with most of the empirical finding that the income elasticity estimates are around one or greater than one (see, e.g., Newhouse (1977), Parkin et al. (1987), and Gerdtham et al. (1992)). Although the value of income elasticity of public health expenditure obtained in our model (0.9692) is higher than the value estimated by Di Matteo and Di Matteo (1998) (0.77), our result is consistent with their conclusion that Canadian public health expenditure is not a luxury good. Our result that income elasticity of private health expenditure exceeds one also receives some empirical support, as studies on some private medical care, such as eyeglasses and plastic surgery, find income elasticities that are substantially greater than one (see e.g., Andersen and Benham (1970), Scanlon (1980), and Parker and Wong (1997)).

In the model economy, private health expenditure and hence the probability of surviving to old adulthood are heterogeneous across individuals. In fact heterogeneity in steady state is one advantage of the model. Most existing studies that model endogenous life-expectancy, such as Blackburn and Cipriani (2002), Chakraborty (2004) and Hall and Jones (2007), do not have disparities in health status across agents. Based on $20,000$ income draws, Figure 1 plots the kernel density and cumulative distribution functions for income, private health expenditure, and survival probability in the steady state. A notable feature from the figure is that the distribution of private health expenditure is much more skewed than the distribution of income, with about 30 percent of the population having zero or close to zero private health expenditure. However, the distribution of survival probabilities appears quite symmetric, due to the contribution of public health. These qualitative features are broadly consistent with the data, though we do not have individual level data to conduct a quantitative comparison.

4.2 Comparative Statics

Next we conduct a numerical exercise to investigate the comparative static properties of the model. The aim is to see how the majority choice of tax rate, which in the model measures the size of public health relative to national income, and the public-private mix of health

Figure 1: Kernel Density and Cumulative Distribution Functions for Income, Private Health Expenditure, and Survival Probability in the Steady State

expenditure respond to variations in the primitives of the model, in particular, how sensitive they are to each variation.

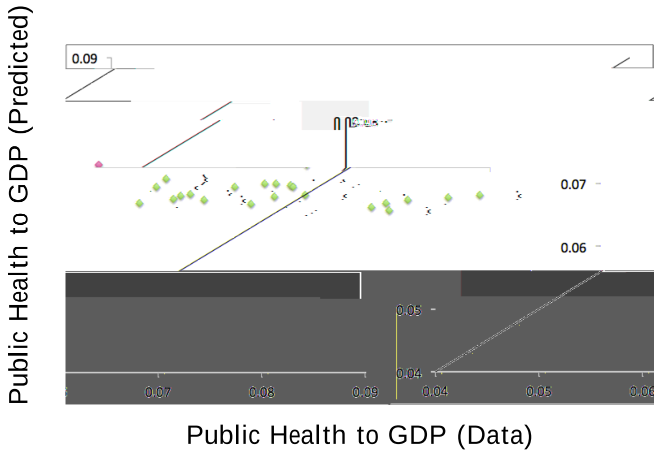
Speci…cally, we examine how the majority choice of tax rate, $_m$, the ratio of median private health to public health expenditure, $h_m=H$, and the share of public health in total health expenditure, $H=(H + h)$, respond to changes in parameters , , , $_H$, , and the statistics that characterise the distribution of income, $y_m=y$ and y. Baseline values for these parameters are the ones described in the calibration above. We consider 5, 10 and 15 percent variations of each parameter around its baseline value, with all other parameters kept at their baseline values. For each variation, $h_m=H$ is determined by (25), and $_m$ is solved from (25) to (27). To get the mean private health expenditure h, we follow the same procedure as in the baseline calibration, then the share of public health in total health spending is calculated as $H=(H + h)$, where H equals the computed $_m$ times y.

Table 2 summarizes the results from the numerical exercise. It is found that the e,

in $_H$, also sensitive to variations in , , and y_m=y, and less sensitive to , , and y. The median survival probability, p_m, decreases with and , and increases with , , $_H$, y_m=y and y. In terms of magnitude, p_m is relatively more sensitive to and y, followed by , , $_H$ and y_m=y, and it is least sensitive to . The ratio of median private health to per capital public health spending, h_m=H, does not vary with , , and y, while decreases with and $_H$ and increases with y_m=y. Again, it is most sensitive to $_H$, and also quite sensitive to variations in and y_m=y. The share of public health in total health expenditure, H=(H + h), increases with , , and $_H$, decreases with , and has no clear relationship to y_m=y and y. It is quite sensitive to $_H$ as well as , while not sensitive to other parameters.

Brief intuitions are as follows. A higher discount factor, , implies that individuals care more about their old-age consumption, so they prefer a lower tax rate in order to have more disposable income to save for old age. As a consequence, the survival probabilities decline. An increase in implies a lower survival probability (at any given level of health capital), so to improve survival probabilities, individuals tend to vote for a higher level of public health. A higher means a higher weight is attached to the health capital in the utility function so that individuals prefer a higher tax rate to achieve a higher level of public health. As a result survival probabilities also increase. A higher implies a greater substitutability between public and private health. With a $_H$ greater than 0:5, individuals tend to substitute private health with public health and vote for a higher tax rate, or in other words, public health crowds out private health when increases. Consequently, the ratio of median private health to public health expenditure falls and the share of public health in total health expenditure rises. A higher $_H$ indicates that the importance of public health rises relative to private health in the formation of health capital and thus, individuals vote for a higher tax rate, which leads to a lower ratio of median private to public health and a higher share of public health.

The relationship between $_m$ and y_m=y is standard: lower income inequality leads to a lower preferred tax rate. Hence, following an increase in y_m=y, the ratio of median private health to public health expenditure increases. The negative effect of y on $_{m}$ confirms the larger sample than used in previous studies, confirm a strong positive effect of income on per capita health expenditure in OECD countries. Ettner (1996) and Di Matteo and Di Matteo (1998) provide country-specific evidence, where the former finds a large positive effect of income on health status in the U.S., and the latter find that one of the key determinants of Canadian per capita provincial government health expenditure is real provincial per capita income. However, there are few empirical studies that investigate how income affects the mixture of public and private health expenditure. One of them is Di Matteo (2000), which studies the public-private mix of health expenditure in Canada and finds that an important determinant of the split is the share of individual income held by the top quintile of the income distribution–a measure of income inequality.In the quantitative exercise, we assume that countries only differ in their average income (y) and ratio of median to mean income (y_m=y



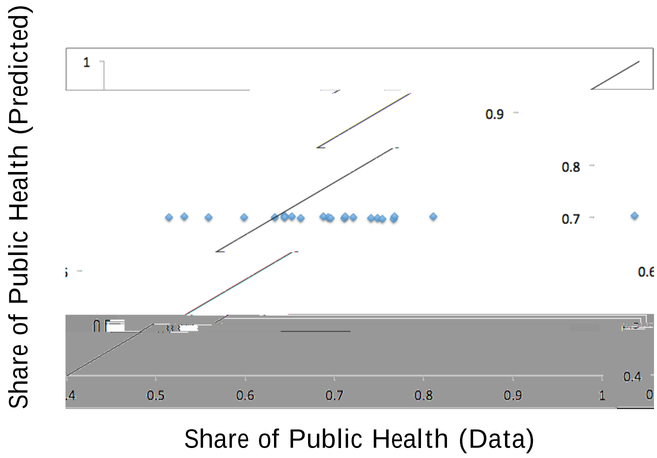


Figure 2: Prediction v.s. Data for a Sample of Democratic Countries

income are closer to their data counterparts and exhibit a bit more variation across countries, suggesting that income distribution might play a minor role in accounting for the size of public health expenditure relative to national income.

These results should be interpreted with caution, since we only consider differences in the mean and variance of income distributions and ignore variations in all other factors across countries. A possible reason for the insignificant role of income distribution is that it is not appropriate to assume that the parameters , , , $_H$, take the same values for each country as Canada. In particular, the comparative static results highlight that the composition of health expenditure is very sensitive to the degree of substitutability between public and private health spending, measured by , and the relative importance of public health, measured by $_H$. Hence, in the next quantitative exercise we calibrate these two parameters for each country in our sample and re-predict the public-private mix of health expenditure.

Another possible explanation is that in the model income is exogenously given such that there is no feedback effects between income and health expenditure: an individual with higher income spends more on health care and higher health spending leads to higher productivity and income.

4.3.2 Public-Private Mix of Health: Model v.s. Data

In the following quantitative exercise we aim to answer the quantitative question outlined in the Introduction: how well does the model explain the observed differences in the public-private mix of health expenditure across countries? That is, we use the model to predict the shares of public health in total health expenditure and compare the predicted values with their data counterparts.

The first quantitative exercise shows that when the parameters (other than y and y_m=y) are assumed to take the same values as Canada for each country, the predicted mixture of health expenditure does not match the data. The comparative static results suggest that the mixture of health expenditure is quite sensitive to

Table 3: Calibrated Values and Computed Share of Public to Total Health Expenditure for Each Country

Country	H		"	$\frac{h_m}{H}$	Model $\frac{H}{H+h}$	Data $\frac{H}{H+h}$	$\frac{\text{Model}}{\text{Data}}$
Australia	0.602	0.024	1.025	0.581	0.6055	0.66725	0.9074
Austria	0.568	0.792	4.808	0.153	0.7888	0.76144	1.0359
Canada	0.58	0.6162	2.606	0.296	0.7027	0.7014	1.0018
Czech Republic	0.502	0.81	5.263	0.453	0.6112	0.8792	0.6951
Denmark	0.557	0.827	5.780	0.190	0.7931	0.835	0.9498
Finland	0.633	0.021	1.021	0.514	0.6357	0.7306	0.8702
France							

Recall that and $_{H}$ capture the interaction of public and private health care in forming the health capital of the society. Our quantitative exercise provides a way to estimate these parameters for each country. The variations in the estimates suggest that the interaction of public and private health care differs substantially across countries in our sample. As there is little empirical work that looks at the interaction between public and private health, we are not able to assess whether our estimates are in line with the existing institutional arrangements of the health care systems in each country. However, there are some country-specific the predicted and actual shares of public health is 0:44 for the whole sample, and is 0:63 if we exclude the U.S. and the Czech Republic from the sample.

We consider the U.S. and the Czech Republic to be outliers in the quantitative analysis.

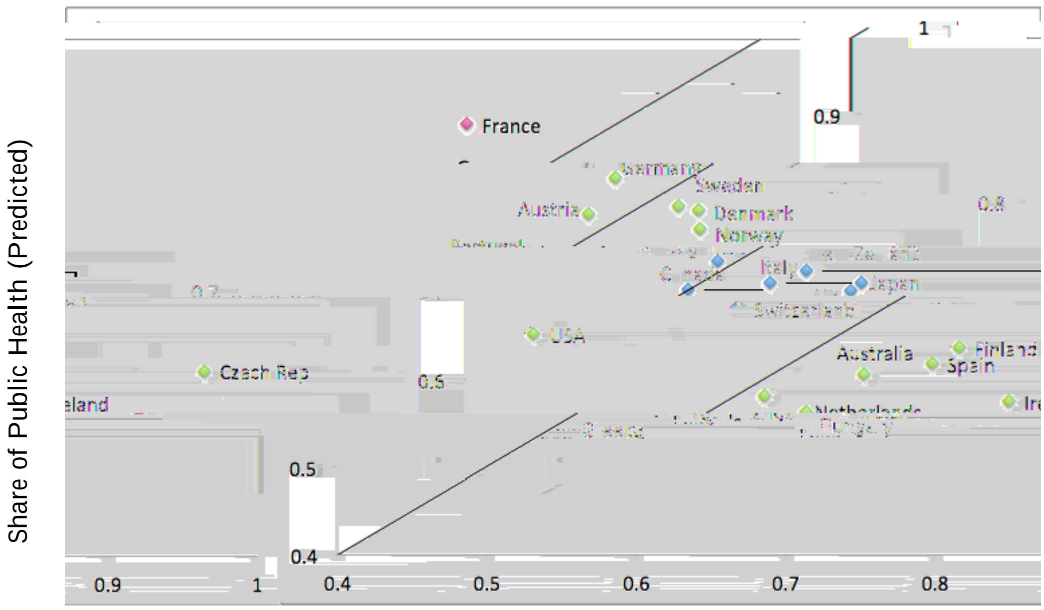


Figure 3: Share of Public Health in a Sample of Democratic Countries

Due to the complexity of the issue, an extensive discussion is beyond the scope of this paper. Nevertheless, next we discuss several factors that may be important for the public-private mix of health expenditure but are not considered in our model, including the demographic structure of the population, the structure of public …nancing and private …nancing of health care, as well as the pricing of health care services.

The demographic structure of the population may have important implications for the composition of health care spending. Health expenditure is typically highly concentrated, in particular, old people account for a much larger fraction of total health spending relative to their share in the population. So an ageing population implies a higher demand for health care services, especially for public long-term health care. In fact, for countries in our sample, most of which have an ageing population, there is a positive correlation (0.22) between the shares of the population above 65 years of age and the shares of public health in total health expenditure.

The focus in this study is on the composition of …nancing of health care: public versus private …nancing. We abstract from the di¤erent types of public and private …nancing of health care, as observed in the data. According to OECD Health Dataset - 2011, government revenues and social insurance are the two main sources of finance for public health care and out-of-pocket and private health insurance are the two main sources of private funding of health care. Figure 4 compares the compositions of public and private financing of health care across countries in the sample. It is clear that the composition of public financing differs substantially across countries. For some countries (e.g., Denmark, Australia, and Ireland) government revenue accounts for more than 95 percent of public financing, while for some other countries (e.g., Netherlands, France and Germany) public health care relies mostly on social insurance based funding. For the financing of private health care, the out-of-pocket contributions range from 30 percent to almost 100 percent. Understanding these differences within each type of financing of health care is important for us to understand the observed differences in the public-private mixture of the overall financing of health care across countries.

Last but not least, the pricing of health care services also has important implications for the mixture of health expenditure. An increase in the prices of private health care services would lead to a higher share of private health expenditure if the demand for private health care is relatively inelastic. Hence differences in the relative prices of health care services across countries, which are well observed in the data (Gerdtham and et al (1992) and Gerdtham and Jonsson (1991)), contribute to the observed differences in the mixture of health expenditure across countries.

5 Conclusions

Despite the large variations in the public-private mix of health expenditure across countries, factors that critically affect the composition of health expenditure have rarely been examined analytically and empirically in the existing literature. In this study, we examined, in the context of a simple overlapping generations model, how the public-private mix of health

spending is determined through majority voting and how this decision is a¤ected by various preference and economic factors. Further, we calibrated the model to conduct a quantitative exercise. The quantitative results are in line with the data, in particular, the predicted mixture of health expenditure matches the data reasonably well for a group of advanced democratic countries, suggesting that the model provides a promising framework to study the choice of public and private spending on health care.

The quantitative exercise also revealed the importance of the degree of substitutability between public and private health and the relative e¤ectiveness of public health vs. private health in explaining the composition of health expenditure, and provided a way to infer these factors from the data. Knowing about these factors can help policy makers to design or reform health care policies to achieve the goals of e¢ ciency and equity in health care …nancing. For instance, if the elasticity of substitution between public and private health is high, i.e., the two types of health expenditure are more substitutable, an increase in one type of health expenditure is more likely to crowd out the other type of health expenditure. So any proposed policy change or reform with respect to the …nancing of health care should take this crowding out e¤ect into consideration. In this respect, our study has important policy implications. To the best of our knowledge, this has not been explored in the existing literature.

As one of the …rst few attempts to formally examine the public-private mix of health care, this study utilizes a simple framework which incorporates voting in a dynamic macro-theoretic model. The model considers several important factors for an individual's decision regarding public and private health spending, such as income, the role of health care in

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Appendix

Proof of Proposition 1. By substituting $p_{m;t} = p(\hat{H}_{m;t-1}) = p(\ \ _m \ \ _{m;t-1})$ into (11), we obtain $\ \ _{m;t} = \ \ _H/[1 + \ \ + \ \ p(\ \ _m \ \ _{m;t-1})]$. Recall that $\ \ _m = [(\ \ _h/\ \ _H)(y_m/y)]^{h}y$, $\partial p_{m;t}/\partial \hat{H}_{m;t-1} > 0$ and $\partial^2 p_{m;t}/\partial \hat{H}^2_{m;t-1} < 0$. Hence, by differentiating $\ \ _{m;t} = \ \ _H/[1 + \ \ + \ \ p(\ \ _m \ \ _{m;t-1})]$ with respect to $\ \ _{m;t-1}$, we obtain

$$\frac{\partial \ \ _{m;t}}{\partial \ \ _{m;t-1}} = - \ \ _H \frac{\ \ _m \frac{\partial p_{m;t}}{\partial \hat{H}_{m;t-1}}}{(1 + \ \ + \ \ p(\ \ _m \ \ _{m;t-1}))^2} < 0,$$

and

$$\frac{\partial^2 \ \ _{m;t}}{\partial \ \ ^2_{m;t-1}} = - \ \ _H \ \ _m \frac{\frac{\partial^2 p_{m;t}}{\partial \hat{H}^2_{m;t-1}}(1 + \ \ + \ \ p(\ \ _m \ \ _{m;t-1})) - 2 \ \ _m \left(\frac{\partial p_{m;t}}{\partial \hat{H}_{m;t-1}}\right)^2}{(1 + \ \ + \ \ p(\ \ _m \ \ _{m;t-1}))^3}$$
$$> 0.$$

Therefore, the the steady state tax rate, $\ \ _m$, is unique.

Next, we show that the steady state tax rate is given by (16). Since $\ \ _{m;t} = \ \ _{m;t-1} = \ \ _m$ at the steady state, by using (15), we obtain the following quadratic function:

$$\ \ ^2_m \ \ _m[(1 + \ \ ) \ \ + \ \ ] + \ \ _m[1 + \ \ (1 - \ \ _H \ \ _m)] - \ \ _H = 0$$

which implies

$$\ \ _m = \frac{-[1 + \ \ - \ \ _H \ \ _m] \pm \sqrt{[1 + \ \ - \ \ _H \ \ _m]^2 + 4[(1 + \ \ ) \ \ + \ \ ] \ \ _H \ \ _m}}{2[(1 + \ \ ) \ \ + \ \ ] \ \ _m}.$$

It is shown below that the positive steady state tax rate is given by:

$$\ \ _m = \frac{-[1 + \ \ - \ \ _H \ \ _m] + \sqrt{[1 + \ \ - \ \ _H \ \ _m]^2 + 4[(1 + \ \ ) \ \ + \ \ ] \ \ _H \ \ _m}}{2[(1 + \ \ ) \ \ + \ \ ] \ \ _m}.$$

$\ \ _m$ has the following properties:

(i) $\ \ _m \in (0; 1)$: The denominator of $\ \ _m$ is positive and so it suffices to show that the numerator of $_m$ is also positive. Since $4[(1 + \ ) \ + \ ] \ _H \ _m > 0$,

$$\sqrt{[1 + \ - \ _H \ _m]^2 + 4[(1 + \ ) \ + \ ] \ _H \ _m} > [1 + \ - \ _H$$

where

$$\tilde{N} \equiv \quad {}_{H} \quad {}_{m}[(1 + \quad) \quad + 2 \quad] + (1 + \quad)^2 > 0.$$

Since @ $_m$=@y$_m$ > 0, and it can be veri…ed that

$$(1 + \quad)^2 \{ \sqrt{[1 + \quad - \quad {}_{H} \quad {}_{m}]^2 + 4[(1 + \quad) \quad + \quad] \quad {}_{H} \quad {}_{m}} \}^2 < (\tilde{N})^2 \, N)^2$$

the following quadratic function:

$$^{2}_{m}\ _{H}y[\ (\ \ + F) + \ \ F] + \ _{m}[\ (1 - \ \ _{H}y) + F] - \ \ = 0$$

which implies

$$_{m} = \frac{-[\ (1 - \ \ _{H}y) + F] \pm \sqrt{[\ (1 - \ \ _{H}y) + F]^{2} + 4\ \ _{H}y[\ (\ \ + F) + \ \ F]}}{2\ _{H}y[\ (\ \ + F) + \ \ F]}.$$

It is shown below that the positive steady state tax rate is given by:

$$_{m} = \frac{-[\ (1 - \ \ _{H}y) + F] + \sqrt{[\ (1 - \ \ _{H}y) + F]^{2} + 4\ \ _{H}y[\ (\ \ + F) + \ \ F]}}{2\ _{H}y[\ (\ \ + F) + \ \ F]}.$$

$_{m}$ has the following properties:

(i) $_{m} \in (0; 1)$: Since the denominator of $_{m}$ is positive, it su¢ ces to show that the numerator of $_{m}$ is also positive. Since $4\ \ _{H}y[\ (\ \ + F) + \ \ F] > 0$, we have

$$\sqrt{[\ (1 - \ \ _{H}y) + F]^{2} + 4\ \ _{H}y[\ (\ \ + F) + \ \ F]} > [\ (1 - \ \ _{H}y) + F],$$

and thus, $_{m} > 0$. To show $_{m} < 1$, it su¢ ces to show that the denominator of $_{m}$ is larger than the numerator of $_{m}$. It is easy to verify that

$$\begin{aligned} & \{2\ _{H}y[\ (\ \ + F) + \ \ F] + [\ (1 - \ \ _{H}y) + F]\}^{2} \\ > \ & \{\sqrt{[\ (1 - \ \ _{H}y) + F]^{2} + 4\ \ _{H}y[\ (\ \ + F) + \ \ F]}\}^{2}, \end{aligned}$$

so the denominator of $_{m}$ is larger than the numerator of $_{m}$:

$$\begin{aligned} & 2\ _{H}y[\ (\ \ + F) + \ \ F] \\ > \ & -[\ (1 - \ \ _{H}y) + F] + \sqrt{[\ (1 - \ \ _{H}y) + F]^{2} + 4\ \ _{H}y[\ (\ \ + F) + \ \ F]}, \end{aligned}$$

and thus, $_{m} < 1$.

(ii) $_{m}$ is locally stable, and converged with damped oscillations: by substituting $_{m;t\square 1} =$

$_m$, $y_{t□1} = y$, and $p_m = \;_m\;_H y/(1 + \;_m\;_H y)$ into $@\;_{m;t}/@\;_{m;t□1}$ obtained above, we have:

$$\left|\frac{@\;_{m;t}}{@\;_{m;t□1}}\right|_{\;_m} = \frac{4\;_H y\; F}{\left\{[\;(1+\;_H y)+F]+\sqrt{[\;(1-\;_H y)+F]^2+4\;_H y[\;(\;+F)+\;F]}\right\}^2},$$

and thus, it su¢ ces to show that the denominator of $|@\;_{m;t}/@\;_{m;t□1}|_{\;_m}$ is larger than the numerator of $|@\;_{m;t}/@\;_{m;t□1}|_{\;_m}$:

$$\left\{[\;(1+\;_H y)+F]+\sqrt{[\;(1-\;_H y)+F]^2+4\;_H y[\;(\;+F)+\;F]}\right\}$$

Since $\partial F/\partial y_m < 0$, and it can be verified that

$$(N_{y_m/y})^2 > \{[\ (\ +F)+\ F](\ +F)+\ \sqrt{[\ (1-\ _H y)+F]^2+4\ _H y[\ (\ +F)+\ F]}\}^2,$$

if $_H$ is large enough such that $_H > 1/(\ y)$. Therefore, $\partial\ _m/\partial y_m > 0$, which implies $\partial\ _m/\partial(y_m/y) > 0$ when private and public health are perfect substitutes.

(iv) $_m$ falls with a rise in y (keeping y_m/y constant): We first differentiate $_m$ in (23) with respect to y:

$$\frac{\partial\ _m}{\partial y} = \frac{[\ (\ +F)+\ F]\{(\ +F)\sqrt{[\ (1-\ _H y)+F]^2+4\ _H y[\ (\ +F)+\ F]}-N_y\}}{2\ _H y^2[\ (\ +F)+\ F]^2\sqrt{[\ (1-\ _H y)+F]^2+4\ _H y[\ (\ +F)+\ F]}}$$

where

$$N_y \equiv (\ (1-\ _H y)+F)(\ _H y+\ (1-\ _H y)+F)+2\ _H y(\ (\ +F)+\ F)>0.$$

Since it can be verified that

$$(\ +F)^2\{[\ (1-\ _H y)+F]^2+4\ _H y[\ (\ +F)+\ F]\}<(N_y)^2,$$

$\partial\ _m/\partial y < 0$. ■

Proof of Single-peaked Preferences for the General Case. From Eq. (24), we find that

$$\frac{\partial h_{i,t}}{\partial\ _t} = -y_{i,t}\frac{\left(\frac{\ _t y_t}{h_{i,t}}\right)^{-1}\frac{y}{y_{i,t}}+1}{\frac{(1+\ p_{i,t}+\ )}{\ }+(1-\ )(1-\ _t)\frac{y_{i,t}}{h_{i,t}}}<0,$$

and

$$\frac{\partial^2 h_{i,t}}{\partial\ _t^2} = \frac{\left\{y_{i,t}(1-\ )\ y\ \left(\frac{h_{i,t}-\ _t\frac{\partial h_{i,t}}{\partial\ _t}}{h_{i,t}^2}\right)\left(\frac{\ _t}{h_{i,t}}\right)^{-1}\right\}\left\{\frac{1}{y_{i,t}}\left(\frac{\ _t}{h_{i,t}}\right)^{-1}\frac{(1+\ p_{i,t}+\ )}{\ }+\ \left(\frac{\ _t}{h_{i,t}}\right)^{-1}\frac{y}{y_{i,t}}+1\right\}}{\left(\frac{(1+\ p_{i,t}+\ )}{\ }+(1-\ )\ y\ \left(\frac{\ _t}{h_{i,t}}\right)\right)^2}>0,$$

where $\ = ((1+\ p_{i,t})/\ )(\ /(1-\ ))$. By differentiating the indirect utility function with

where $= ((1 + \quad p_{i;t}) \quad) = \quad + 1$, and

$$= (\quad - 1)(\quad h_{i;t})^2 + \quad y_{i;t} \;\; {}_t \;\; h_{i;t}(\quad - 2) + (\quad y_{i;t})^2 (1 - \quad {}_t) \quad {}_t .$$

So $@^2 V_{it} = @ \; {}^2_t < 0$, and hence the voters' preferences are single-peaked. ■

Proof of $@ \; {}_m = @y < 0$ for the General Case. From (26), de…ne the function at the steady state as Υ:

$$\Upsilon = \frac{1 - \quad {}_m}{{}_m} \frac{y_m}{y} \frac{H}{h_m} - 1 - \frac{1 + \quad p_m}{} \left[1 + \frac{{}_H}{{}_h} \left(\frac{H}{h_m} \right) \right] = 0,$$

where

$$p_m = p\left(\hat{H}_m\right) = \frac{1}{+ \frac{1}{\left[{}_H + {}_h \left(\frac{h_m}{H} \right)^{\frac{}{1-}} \right] \; {}_m y}}.$$

Thus,

$$\frac{@p_m}{@ \; {}_m} = \frac{C_1 \frac{1}{y} \left(\frac{1}{{}_m} \right)^2}{\left(\; + \frac{C_1}{H} \right)^2}, \text{ and } \frac{@p_m}{@y} = \frac{C_1 \frac{1}{{}_m} \left(\frac{1}{y} \right)^2}{\left(\; + \frac{C_1}{H} \right)^2},$$

where $C_1 = 1 = \left({}_H + {}_h (h_m = H)^{\frac{}{1-}} \right)$. By implicit function theorem, we thus obtain

$$\frac{@ \; {}_m}{@y} = - \frac{\frac{@}{@y}}{\frac{@}{@ \; {}_m}} = - \frac{- - \left[1 + \frac{{}_H}{{}_h} \left(\frac{H}{h_m} \right) \right] \frac{@p_m}{@y}}{- \frac{1}{{}^2_m} \frac{y_m}{y} \frac{H}{h_m} - - \left[1 + \frac{{}_H}{{}_h} \left(\frac{H}{h_m} \right) \right] \frac{@p_m}{@ \; {}_m}} < 0,$$

keeping $y_m = y$ unchanged, and the fact that $H = h_m$ is independent of the tax rate implied by (25). ■